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# Long-range dependence in returns and volatility of Central European Stock Indices

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## **Abstract:**

In the paper, we research on the presence of long-range dependence in returns and volatility of BUX, PX and WIG between years 1997 and 2009 with use of classical and modified rescaled range. Moving block bootstrap with pre-whitening and post-blackening is used for the construction of confidence intervals for the hypothesis testing. We show that there is no significant long-range dependence in returns of all examined indices. However, significant long-range dependence is detected in volatility of all three indices. The results for returns are contradictory with several studies which claim that developing markets are persistent. However, majority of these studies either do not use the confidence intervals at all or only the ones based on standard normal distribution. Therefore, the results of such studies should be reexamined and reinterpreted.

**Keywords:** long-range dependence, rescaled range, modified rescaled range, bootstrapping

**JEL:** C4, C5, G15

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# 1 Introduction

Long-range dependence was examined and claimed to be found in several different assets such as stock indices (Di Matteo et al., 2003, 2005; Peters, 1994; Matos et al., 2008), interest rates (Di Matteo, 2007; Cajueiro and Tabak, 2007), government bonds (Carbone et al., 2004; Di Matteo et al., 2005), exchange rates (Vandewalle et al., 1997), electricity prices (Alvarez-Ramirez and Escarela-Perez, 2009) and commodities (Power and Turvey, 2010) using various methods. Majority of research papers interpret long-range dependence on the basis of comparison of long-range dependence characteristic parameter - Hurst exponent  $H$  - with the critical value of 0.5. Such approach without further discussion is questionable as a significant part of the most popular Hurst exponent  $H$  estimation techniques are biased by the presence of short-term memory, e.g. *ARIMA* or (*G*)*ARCH*, in the underlying process. However, distinguishing between short and long-range dependence is essential as both types have different implications for important financial areas such as portfolio selection, option pricing and risk management.

To deal with the problem, we use the classical and modified rescaled range analysis (Hurst, 1951; Lo, 1991) as well as moving block bootstrap with pre-whitening and post-blackening procedure to construct confidence intervals for testing the hypothesis of no long-range dependence (Srinivas and Srinivasan, 2000).

In the paper, we focus on detection of long-range dependence in returns and volatility of the stock indices of the Czech Republic (PX), Hungary (BUX) and Poland (WIG). The data set covers the evolution for the periods of 26.7.2001 - 30.6.2009 for BUX, 7.7.1997 - 30.6.2009 for PX and 14.10.2003 - 30.6.2009 for WIG.

The paper is structured as follows. In Section 2 and 3, long-range dependence is shortly introduced together with rescaled range and modified rescaled range analysis. Section 4 describes the research methodology with focus on moving block bootstrap and choice of its parameters. In Section 5, the data set is briefly examined. Section 6 presents the results and asserts that there is no long-range dependence in returns of any examined index whereas there is strong long-range dependence in volatility of all examined indices. Section 7 concludes.

## 2 Long-range dependence

Long-range dependence is present in the stationary time series if the autocorrelation function  $\rho$  of said process decays as  $\rho(k) \approx Ck^{2H-2}$  for lag  $k$  approaching infinity. Parameter  $0 < H < 1$  is called Hurst exponent after water engineer Harold Edwin Hurst who used the exponent for description of river flows behavior (Hurst, 1951; Mandelbrot and van Ness, 1968).

The critical value of Hurst exponent is 0.5 and suggests two possible processes.  $H$  being equal to 0.5 implies either an independent process (Beran, 1994) or a short-term dependent process (Lillo and Farmer, 2004). Independen-

dent process has zero auto-covariances at all non-zero lags. On the other hand, short-term dependent process shows non-zero auto-covariances at low lags which decay exponentially to zero at high lags.

If  $H > 0.5$ , the auto-covariances of the process are positive at all lags so that the process is called long-range dependent with positive correlations (Embretchts and Maejima, 2002) or persistent (Mandelbrot and van Ness, 1968). The auto-covariances are hyperbolically decaying and non-summable so that  $\sum_{k=0}^{\infty} \gamma(k) = \infty$  (Beran, 1994). On the other hand, if  $H < 0.5$ , the auto-covariances are significantly negative at all lags and the process is said to be long-range dependent with negative correlations (Embretchts and Maejima, 2002) or anti-persistent (Mandelbrot and van Ness, 1968). Similarly to the previous case, the auto-covariances are hyperbolically decaying but summable so that  $0 < |\sum_{k=0}^{\infty} \gamma(k)| < \infty$  (Embretchts and Maejima, 2002). The persistent process implies that a positive movement is statistically more likely to be followed by another positive movement or vice versa. On the other hand, the anti-persistent process implies that a positive movement is more statistically probable to be followed by a negative movement and vice versa (Vandewalle et al., 1997).

### 3 Classical and modified rescaled range analysis

Rescaled range analysis ( $R/S$ ) is the oldest Hurst exponent estimation method and was proposed by Harold E. Hurst while working as an engineer in Egypt (Hurst, 1951) and further adjusted by Mandelbrot and Wallis (1968). In the procedure, one divides the time series of  $T$  continuous returns into  $N$  adjacent sub-periods of length  $v$  so that  $Nv = T$ . For each sub-period, the rescaled range of the profile (cumulative deviations from the mean) is calculated as  $R_i/S_i$ , where  $R_i$  is a range of the corresponding profile and  $S_i$  is a standard deviation of corresponding returns. The same procedure is applied to each sub-period of given length and average rescaled range is calculated (Peters, 1994). Rescaled ranges then scale as

$$(R/S)_v \approx cv^H$$

with varying  $v$  where  $c$  is a constant (Taqqu et al., 1995). The linear relationship in double-logarithmic scale indicates the power scaling (Weron, 2002). To uncover the scaling law, an ordinary least squares regression on logarithms of each side of the equation is applied and  $H$  is estimated.

$V$  statistic, which is used for cycles detection, stability testing of Hurst exponent or change in scaling behavior (crossover) detection, is defined as

$$V_v = \frac{(R/S)_v}{\sqrt{v}}$$

and converges to distribution defined as  $F_V(x) = 1 + 2 \sum_{k=1}^{\infty} (1 - 4k^2 x^2) e^{-2(kx)^2}$  for an independent process (Lo, 1991; Hurst, 1951; Peters, 1994). The statistic is constant, increasing and decreasing with increasing scale  $v$  for no long-range dependence, persistence and anti-persistence, respectively.

As  $R/S$  analysis presented above (usually called classical) is biased by presence of short-term memory, Lo (1991) proposed modified rescaled range analysis ( $M - R/S$ ) which differs from the classical one by the use of modified standard deviation defined with a use of auto-covariance of the original series  $\gamma_j$  in the selected sub-interval up to lag  $\xi$  as

$$S_{I_n}^M = \sqrt{S_{I_n}^2 + 2 \sum_{j=1}^{\xi} \gamma_j \left(1 - \frac{j}{\xi + 1}\right)}.$$

Thus,  $R/S$  turns into a special case of  $M - R/S$  with  $\xi = 0$ . Choice of the correct lag  $\xi$  is critical for the estimation of modified rescaled ranges (Wang et al., 2006; Teverosky et al., 1999). Lo (1991) suggested optimal lag based on the first-order autocorrelation coefficient of the original series  $\rho(1)$  defined as (where  $\lfloor \cdot \rfloor$  is the nearest lower integer operator)

$$\xi^* = \left\lceil \left[ \left(\frac{3v}{2}\right)^{\frac{1}{3}} \left(\frac{2\rho(1)}{1 - \rho(1)^2}\right)^{\frac{2}{3}} \right] \right\rceil. \quad (1)$$

One then gets estimates of modified rescaled ranges for the set scale  $v = T$ , constructs the  $V$  statistics and compares it with critical values of above shown distribution  $F_V$  with null hypothesis of no long-range dependence.

## 4 Research methodology

### 4.1 Moving block bootstrap

Bootstrap method (Efron, 1979) was developed to deal with statistical properties of small samples. The basic notion behind the procedure is reshuffling of the original series and repeated estimation of a specific parameter or statistic. By shuffling, the distribution of original series remains unchanged while the possible dependencies are distorted (Davison and Hinkley, 1997). Hypothesis can be then tested on confidence intervals based on bootstrapped estimates. For our purposes, the simple bootstrap is not enough as the shuffling rids us not only from long-range dependence but the short-range one as well. Srinivas and Srinivasan (2000) proposed a modified method which retains the short dependence characteristics but lacks the long one - moving block bootstrap with pre-whitening and post-blackening.

In the procedure, the time series  $\{x_t\}_{t=1}^T$  is firstly pre-whitened by a specific process - usually  $AR(p)$  - and residuals  $\varepsilon_t$  are obtained. These are further centered around average value  $\bar{\varepsilon}$  so that the centered residuals  $\{\varepsilon_t^c\}_{t=1}^T$  are defined as  $\varepsilon_t^c = \varepsilon_t - \bar{\varepsilon}$ . The series  $\{\varepsilon_t^c\}_{t=1}^T$  is then divided into  $m$  blocks of length  $\zeta$  while  $m\zeta = T$ . The blocks are then reshuffled and post-blackened with the use of the model from the pre-whitening part and residuals  $\varepsilon_t^c$  to form new bootstrapped time series  $\{x_t^b\}_{t=1}^T$ . Such time series retains the short-range dependence, potential heteroskedasticity and trends as well as the distribution of the original

time series. However, for small enough  $\zeta$ , the long-range dependencies are torn. Such method is advantageous compared to simple moving block bootstrap as it retains the potential short-range dependence better.

The examined statistic is then estimated on the new time series. The procedure is repeated  $B$  times so that the confidence intervals for hypothesis testing of no long range dependence can be constructed. The estimate of the statistic of the original time series is then compared with the confidence intervals and the hypothesis is either rejected or not.

## 4.2 Parameters choice

As it is a presence of long-range dependence in the process rather than its specific form or level which is important for financial implications, we estimate  $V$  statistics derived from  $R/S$  and  $M - R/S$  rather than Hurst exponent  $H$ . Such procedure is used for several reasons. Firstly, there are many different methods for Hurst exponent estimation and there is no consensus which one is the best - for comparison, see [Weron \(2002\)](#); [Couillard and Davison \(2005\)](#); [Grech and Mazur \(2005\)](#); [Peters \(1994\)](#); [Alessio et al. \(2002\)](#); [Carbone et al. \(2004\)](#). Secondly, Hurst exponent  $H$  based on different methods is only an estimate of real Hurst exponent and has quite wide confidence intervals for finite samples ([Weron, 2002](#); [Kristoufek, 2009](#); [Couillard and Davison, 2005](#)). And thirdly, Hurst exponent estimation usually assumes the underlying process to be either fractional Brownian motion ( $fBm$ ) or from family of autoregressive fractionally integrated moving averages ( $ARFIMA$ ) or from family of fractionally integrated (generalized) autoregressive conditional heteroskedasticity models ( $FI(G)ARCH$ ). However, the complex dynamics of the underlying process can be more complicated.

For moving block bootstrap, we use  $AR(1)$  process for pre-whitening and post-blackening and  $\zeta = 10$ . The lengths of the time series  $T$  are then taken as a multiple of 10. Such choice should be sufficient for ridding of long-range dependence while the other properties remain similar to the original process. For each time series, we construct 2.5% and 97.5% confidence intervals from corresponding quantiles of 1000 bootstrapped time series ( $B = 1000$ ). The procedure is repeated for lags  $\xi = 0, 1, \dots, 10$  while  $\xi \in \mathbb{Z}_0^+$ .

## 5 Data

We apply classical and modified rescaled range on returns and volatility of BUX, PX and WIG. Let  $P_{i,t}$  be a closing value of index  $i$ , for  $i = 1, 2, 3$  with respect to three examined indices, at time  $t$ , for  $t = 0, \dots, N_i$  where  $N_i$  is time series length of index  $i$ . Continuous returns of index  $i$  at time  $t$  are then defined as  $r_{i,t} = \log(P_{i,t}/P_{i,t-1})$  for  $t = 1, \dots, N_i$ . As a measure of volatility, we use absolute returns defined as  $|r_{i,t}|$  for index  $i$  at time  $t$ .

The examined period is the longest for PX starting from 7.7.1997, followed by BUX starting from 26.7.2001 and the shortest sample is the one of WIG from

14.10.2003. The last examined trading day for all three indices is 30.6.2009 so that the current financial crisis is included<sup>1</sup>. The evolution of index values and corresponding returns is presented in Figure 1. Basic descriptive statistics are summed in Table 1. From the table, we can see that the returns are in hand with basic risk/return notion of the financial theory - PX is the least risky but with the lowest average return whereas WIG offered the highest average return with the highest risk measured by standard deviation. All returns are negatively skewed and leptokurtic and thus not normally distributed according to Jarque-Bera statistic as well (Jarque and Bera, 1981). The result is confirmed by QQ-plots, which are shown together with histograms in Figure 2. Basic descriptive statistics are in hand with the stylized facts of the financial markets (Cont, 2001). WIG seems to be closest to normality when compared to the other two indices. Further, stationarity cannot be rejected by KPSS test (Kwiatkowski et al., 1992) for any examined index.

The results of the basic statistical analysis uncover the importance of bootstrap method for construction of confidence intervals opposed to the ones based on standard normal distribution such as in Weron (2002). Methods can be sensitive to different distributions and bootstrapping avoids such pitfall.

## 6 Results and discussion

Table 2 and Figure 3 sum the estimated  $V$  statistics for both  $R/S$  (with  $\xi = 0$ ) and  $M - R/S$  for the whole examined period together with confidence intervals based on moving block bootstrap. Table 2 also shows the estimates for the optimal lag  $\xi^*$  based on Equation 1 in bold font. For returns, the optimal lag was proposed as 4th, 5th and 2nd for BUX, PX and WIG, respectively. Similarly for volatility, the optimal lag was estimated as 10th, 12th and 4th for BUX, PX and WIG, respectively. Such propositions indicate that there is the strongest short-range dependence in PX and quite weak one in WIG.

The estimates for all examined lags  $\xi$  based on both methods show very homogeneous results - on one hand, there is no long-range dependence in returns of BUX, PX and WIG and on the other hand, there is long-range dependence in volatility for all examined indices. Such outcomes put some previously cited results into question since the indices of the Central Europe are usually marked as the less developed and less efficient markets. For such markets, persistence was usually claimed to be found. However, majority of studies do not take the statistical properties of the tested series into consideration and only compare the estimates of Hurst exponent  $H$  or  $V$  statistic with the critical values. Therefore, the results should be reexamined with respect to the statistical properties of the series as well as possible short-range dependence.

Last but not least, Figure 3 also compares the confidence intervals based on bootstrapping with the ones based on Lo (1991) which are correct given the fact that the lag  $\xi$  was chosen correctly. The bootstrapped confidence intervals based on the time series of returns are approximately equal to the ones of Lo

<sup>1</sup>All available data were obtained from [www.dukascopy.com](http://www.dukascopy.com)



(1991) for all examined lags. As this is true even for  $\xi = 0$ , the short-range dependence is either not present at all or very weak and does not bias the estimates of classical  $R/S$ . The situation is more interesting for volatility. The bootstrapped confidence intervals do not collide enough up till between lags 8 and 9. For lower lags  $\xi$ , both bootstrapped critical values are much higher than the ones of Lo (1991). The optimal lag is then not the one of Equation 1 but the one where the confidence intervals based on bootstrapping and the ones based on Lo (1991) equal.

Moreover, the results rise a serious question whether  $M - R/S$  needs to be used at all since the confidence intervals based on the moving block bootstrap implied the same interpretation for both  $R/S$  and  $M - R/S$ . Using  $R/S$ , we reject the null hypothesis of no long-range dependence for the volatility and not for the returns. The same is true for  $M - R/S$ . Therefore, it is rather the use of the moving block bootstrap than the use of  $M - R/S$  which gives the crucial information for the hypothesis testing.

Nevertheless, the difference between the two gives us some additional information. For BUX and PX, the volatility is long-range dependent as well as short-range dependent with  $AR(1)$  being the correct choice. WIG, on the other hand, showed the lowest level of short-range dependence, which is reflected in the lowest departure from the confidence intervals of Lo (1991), but still has the highest estimates of  $V$  statistics for all lags  $\xi$  compared to the other two. Such result implies two possible interpretations - either the short-range dependence should be modeled by some other model or the long-range dependence is stronger in volatility of WIG than it is in volatility of BUX and PX. The latter option seems to be more rational as the difference between the two types of confidence intervals is the lowest for WIG as it was mentioned earlier.

The results imply that the processes can be modeled with a use of nonlinear models. As there is no long-range dependence found in the returns,  $ARFIMA$  models are not useful. However, the short and long-range dependence in volatility implies that e.g.  $FIGARCH$  models can be used with better results than simple  $GARCH$ , which covers the short-range dependence only. The persistence of the volatility shows that the shocks in magnitude decay hyperbolically, which is slower than implied by  $GARCH$  models.

## 7 Conclusion

We researched on the presence of long-range dependence in returns and volatility of BUX, PX and WIG between years 1997 and 2009. From several techniques for long-range dependence detection, we chose classical and modified rescaled range and tested the long-range dependence on the basis of  $V$  statistics. To avoid the potential short-range dependence, distributional, heteroskedasticity and trend complications, we applied moving block bootstrap with pre-whitening and post-blackening for the construction of confidence intervals for the hypothesis testing. On one hand, we showed that there was no significant long-range dependence in returns of all examined indices. On the other hand, significant long-range

dependence was detected in volatility for all three indices. For BUX and PX, both significant short and long-range dependence was detected whereas for WIG, short-range dependence was only weak but the long-range dependence was the strongest from the three indices.

Moreover, we have discussed on possibility of finding the optimal lag for the modified rescaled range procedure. The results for returns are contradictory with several studies which claim that developing markets are persistent. However, majority of these studies either do not use the confidence intervals at all or only the ones based on standard normal distribution. Therefore, the results of such studies should be reexamined and reinterpreted.

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Table 1: Descriptive statistics of BUX, PX and WIG returns

	BUX	PX	WIG
<b>Mean</b>	0,000438	0,0002	0,00016
<b>Min</b>	-0,12649	-0,16185	-0,08443
<b>Max</b>	0,13178	0,12364	0,08155
<b>Standard deviation</b>	0,01682	0,01535	0,01688
<b>Skewness</b>	-0,22025	-0,48709	-0,2773
<b>Excess kurtosis</b>	7,14249	12,27884	2,7021
<b>Jarque-Bera statistic</b>	4160,749	18863,73	465,702
<b>- p-value</b>	0,0000	0,0000	0,0000
<b>KPSS</b>	0.3996	0.2240	0.4677
<b>Observations</b>	1950	2980	1460
<b>Start date</b>	26.7.2001	7.7.1997	14.10.2003
<b>End date</b>	30.6.2009	30.6.2009	30.6.2009

Table 2:  $V$  statistics and bootstrapped confidence intervals

	$\xi$	returns			volatility		
		$V$	$P_{97.5}$	$P_{0.25}$	$V$	$P_{97.5}$	$P_{0.25}$
<b>BUX</b>	0	1.7978	1.9076	0.8578	5.1971*	3.3785	1.5066
	1	1.7319	1.8909	0.8052	4.5694*	2.9306	1.3111
	2	1.7488	1.8683	0.8172	4.1498*	2.6644	1.1927
	3	1.7696	1.9243	0.8399	3.8433*	2.4675	1.0984
	4	<b>1.7556</b>	<b>1.9229</b>	<b>0.8457</b>	3.6085*	2.3184	1.0622
	5	1.7341	1.8685	0.8496	3.4136*	2.3168	1.0174
	6	1.7231	1.8355	0.8375	3.2356*	2.1442	0.9917
	7	1.7314	1.8547	0.8477	3.0838*	2.1131	0.9540
	8	1.7327	1.9073	0.8309	2.9545*	2.0287	0.9056
	9	1.7245	1.8843	0.8505	2.8462*	1.9742	0.9108
	10	1.7175	1.8588	0.8442	<b>2.7514*</b>	<b>2.0329</b>	<b>0.8636</b>
<b>PX</b>	0	1.8083	1.8146	0.8292	5.3791*	3.5733	1.5431
	1	1.7313	1.7676	0.7809	4.6937*	3.0088	1.2965
	2	1.7274	1.8226	0.8070	4.2041*	2.7325	1.2158
	3	1.7344	1.8025	0.7861	3.8634*	2.5371	1.1483
	4	1.7273	1.7825	0.8025	3.6027*	2.4011	1.0632
	5	<b>1.7164</b>	<b>1.8606</b>	<b>0.7957</b>	3.3873*	2.3298	0.9863
	6	1.7099	1.7943	0.7817	3.2108*	2.2459	0.9581
	7	1.7105	1.8159	0.7980	3.0608*	2.1473	0.9295
	8	1.7085	1.7869	0.8021	2.9330*	2.0862	0.8686
	9	1.7082	1.8393	0.8195	2.8223*	1.9996	0.9090
	10	1.7029	1.8544	0.7954	2.7237*	2.0228	0.8651
<b>WIG</b>	0	1.7275	1.8327	0.8266	5.5596*	2.9259	1.2842
	1	1.6942	1.7381	0.8059	5.2617*	2.8235	1.1756
	2	<b>1.7008</b>	<b>1.7774</b>	<b>0.7801</b>	4.9856*	2.5225	1.1301
	3	1.6937	1.7514	0.7500	4.6606*	2.4410	1.0539
	4	1.6894	1.7597	0.8057	<b>4.3816*</b>	<b>2.3968</b>	<b>1.0051</b>
	5	1.6900	1.7238	0.7913	4.1307*	2.2174	0.9584
	6	1.6892	1.7578	0.7737	3.9156*	2.1753	0.9465
	7	1.6971	1.7453	0.7660	3.7368*	2.1025	0.9068
	8	1.7042	1.7611	0.7955	3.5852*	2.0843	0.9252
	9	1.7043	1.7373	0.7570	3.4458*	2.0804	0.8959
	10	1.7037	1.7659	0.8005	3.3211*	1.9788	0.8713

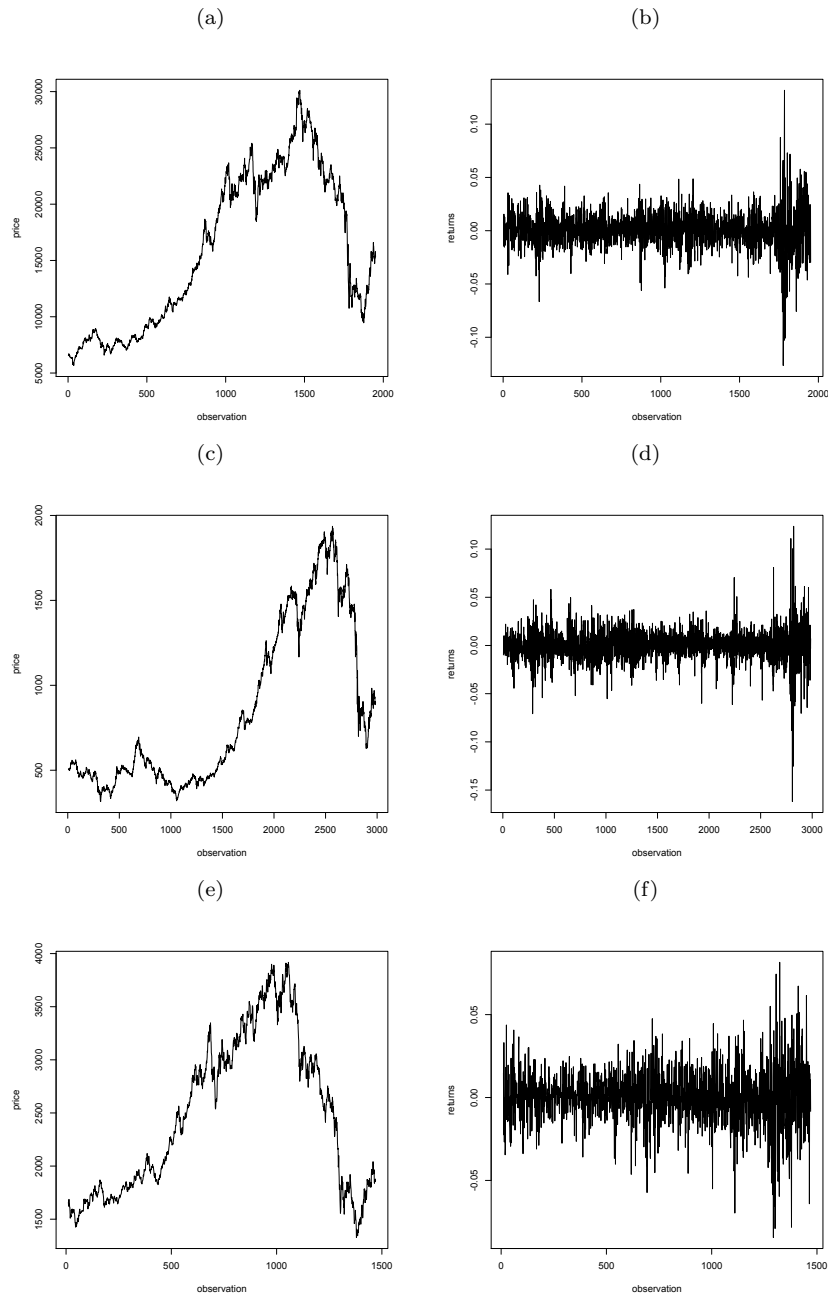


Figure 1: Evolution of index values and returns for BUX, PX and WIG. BUX, PX and WIG are shown on the first, second and third row, respectively, with prices in the first column and returns in the second one. BUX is shown for the period 26.7.2001 - 30.6.2009, PX for the period 7.7.1997 - 30.6.2009 and WIG for the period 14.10.2003 - 30.6.2009.

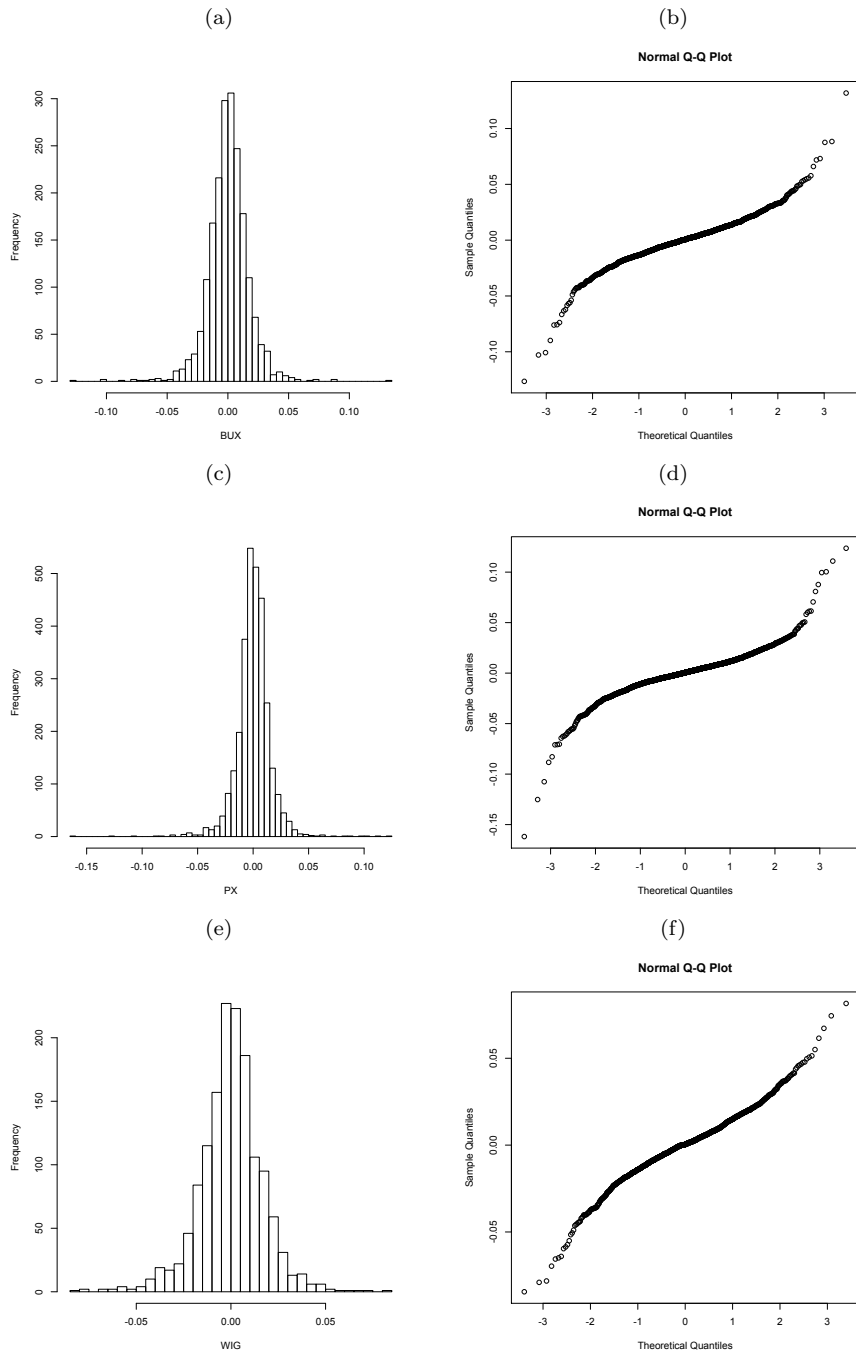


Figure 2: Histograms and QQ-plots for BUX, PX and WIG returns. BUX, PX and WIG are shown on the first, second and third row, respectively, with histograms in the first column and QQ-plots in the second one. QQ-plots compare the quantiles of respective returns with the quantiles of the normal distribution.



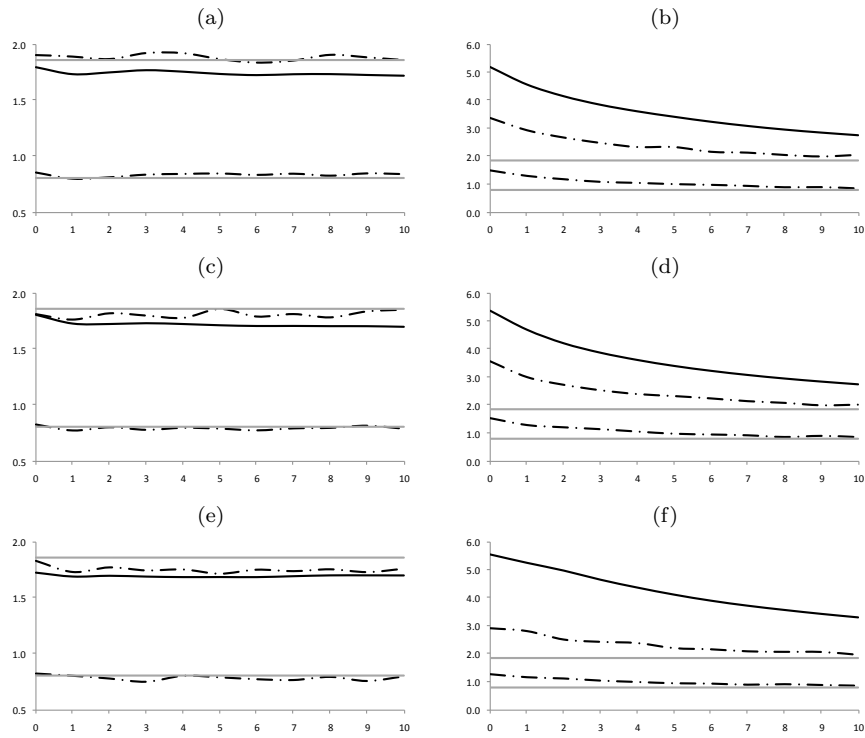


Figure 3: Estimates of  $V$  statistics for returns (first column) and volatility (second column) for BUX (first row), PX (second row) and WIG (third row). The x-axis represents the lag  $\xi$  used. Solid black curve represents the actual estimates of  $V$  statistics, dashed black curves show the 2.5% and 97.5% confidence intervals based on moving block bootstrap procedure and grey solid lines represent the 2.5% and 97.5% confidence intervals of Lo (1991).

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